Letters

The Exact Numerical Evaluation of the Complex Dielectric Constant of a Dielectric Partially Filling a Waveguide

P. I. SOMLO

Abstract—The simultaneous, complex, transcendental equations linking the dielectric constant of a slab partially filling a rectangular waveguide, and the propagation constant of the composite guide have been solved by iteration. Typical results of $\epsilon = f(\gamma)$ are presented.

A recent communication [1] advocated the determination of the complex dielectric constant of solid samples by placing a slab of the sample material of guide width, but of less than guide height, in a rectangular waveguide slotted line and measuring the guide wavelength and attenuation in the composite section, with a short-circuit termination. Calculating the cutoff condition of the composite guide by the transverse resonance method, the propagation constants of the composite guide and those of the air- and dielectric-filled regions may be linked by the transcendental equation:

$$\epsilon h \tan h(b-d) = -l \tan ld \qquad (1)$$

with the additional links of $\gamma^2 - (\pi/a)^2 = h^2 - k_0^2 = l^2 - \epsilon k_0^2$, where a is the width of the guide; b the height; d the thickness of the dielectric having a dielectric constant ϵ; h and l are the propagation constants of the air- and dielectric-filled regions, respectively; γ is the measured propagation constant of the composite guide; and $k_0 = 2\pi/\lambda_0$. Approximate solutions for low-loss materials have been published recently [1], [2] for the fundamental mode of propagation.[1]

It is the purpose of this letter to report that exact solutions of (1) have been obtained by an iterative procedure in the complex domain using a small digital computer (Supernova 8K). On entering the guide dimensions, the thickness of the slab, the frequency, the measured loss per unit length, and the measured guide wavelength, (1) is solved for l, which together with the additional links results in $\epsilon = \epsilon' - j\epsilon''$. Due to the periodic nature of the tangent function in (1), care has to be exercised to reject higher order solutions. However, once a "realistic" solution has been found, it is used as the starting point for the iteration of "neighboring" solutions for different data.

Because of the large number of parameters involved, only a few typical plots of $\epsilon = f(\gamma)$ are given in Fig. 1 for X-band waveguide (WG 16) at 10 GHz, for three different thicknesses of samples, resting on the broad wall of the guide. The regions of the conformal maps having $\epsilon' < 1$ are nonrealizable conditions, and these combinations of α and λ_g would never be encountered measuring natural dielectric materials.

In carrying out the measurement as suggested [1], care has to be exercised that higher order modes are not propagating (this condition may be checked by taking a plot of the VSWR pattern), and that the length of the dielectric sample is such that its ends are sufficiently far from the regions of probe travel for the evanescent modes set up at the ends of the slab to have died down sufficiently.

The computer program has been written for the more general case of the dielectric slab raised from the broad wall by distance t, in which case the transcendental equation is

$$\epsilon h \tan h(b-d-t) + l \frac{\epsilon h \tan ht + l \tan ld}{l - \epsilon h \tan ht \tan ld} = 0. \qquad (2)$$

Manuscript received July 3, 1973.

The author is with the Division of Applied Physics, National Standards Laboratory, Sydney, Australia.

[1] These references and the present work assume that the waveguide is loss free.

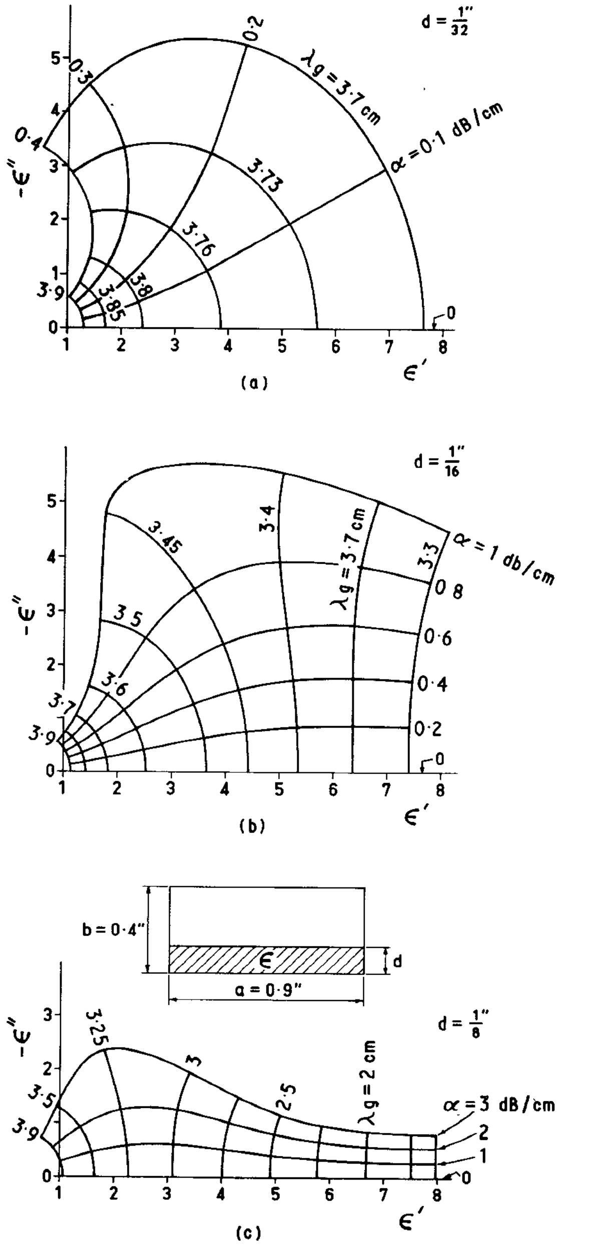

Fig. 1. Plots of $\epsilon = f(\gamma)$ at 10 GHz, in X-band waveguide (WG 16), for three different thicknesses of dielectric sample, resting on the broad wall, i.e., $t = 0$. (a) $d = 1/32$ in. (b) $d = 1/16$ in. (c) $d = 1/8$ in.

It is evident that in the case of the sample resting on the broad wall, i.e., $t = 0$ or $t = b - d$, (2) simplifies to (1).

Fig. 2 illustrates the effects on the measured loss and guide wavelength, of raising a typical sample from the broad wall. The maximum slopes of α and λ_g at $t = 0$ confirm the warning [1] that there should be no air gaps when the sample is placed against the broad wall. As expected from symmetry, least sensitivity to t is shown when the sample is centered at half-guide height, but this results in the smallest changes in α and λ_g as compared to the empty guide, therefore higher measurement resolution is required. However, for slightly warped samples it is the recommended method.

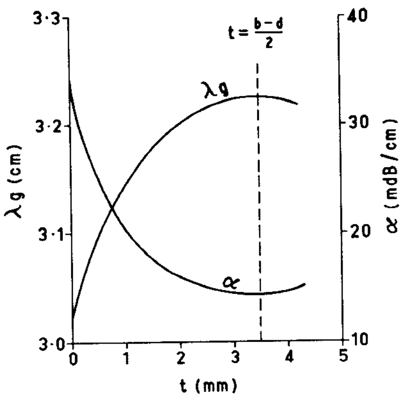

Fig. 2. The effects on α and λ_g of the composite guide of separating the sample from the broad wall by distance t, for a typical sample of $\epsilon = 3 - j0.02$, 1/8 in. thick, measured at 10 GHz, in WG 16 guide.

The computer program, written in Basic, is not claimed to have been optimized for the speed of approach to the root (Newton's method is used to approach the solution along the direction of the steepest descent). This program is available on request.

REFERENCES

[1] P. Bhartia and M. A. K. Hamid, "Dielectric measurements of sheet materials," *IEEE Trans. Instrum. Meas.* (Short Papers), vol. IM-22, pp. 94–95, Mar. 1973.

[2] V. R. Bui and R. R. J. Gagne, "Dielectric losses in an *H*-plane-loaded rectangular waveguide," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-20, pp. 621–623, Sept. 1972.

The Impedance and Scattering Properties of a Perfectly Conducting Strip Above a Plane Surface-Wave System

JOHN BROWN

***Abstract*—Gillespie and Kilburg have postulated that the fraction of the incident surface-wave power radiated by a conducting strip does not exceed 0.5. This result is shown to be a consequence of the representation of the strip by a shunt impedance.**

With the notation used by Gillespie and Kilburg [1], the fraction of the incident power radiated by the strip is

$$P_{\text{rad}} = \text{Re}\,|\,1 + \Gamma\,|^2/\bar{Z}.$$

From [1, eq. (4)],

$$\Gamma = -1/(1 + 2\bar{Z})$$
$$= -\bar{Y}/(2 + \bar{Y})$$

where $\bar{Y} = 1/\bar{Z}$. Hence

$$P_{\text{rad}} = \text{Re}\, 4\bar{Y}/\,|\,2 + \bar{Y}\,|^2$$
$$= \frac{4g}{(2+g)^2 + b^2}$$

when $\bar{Y} = g + jb$.

The maximum value of P_{rad} is seen to occur when $g = 2$ and $b = 0$ and to have the value

$$P_{\text{rad,max}} = 0.5$$

as postulated.

REFERENCES

[1] E. S. Gillespie and F. J. Kilburg, "The impedance and scattering properties of a perfectly conducting strip above a plane surface-wave system," *IEEE Trans. Microwave Theory Tech.* (Short Papers), vol. MTT-21, pp. 413–419, June 1973.

Manuscript received September 4, 1973.
The author is with the Department of Electrical Engineering, Imperial College of Science and Technology, London, England.

Comments on "Measured Noise Temperature Versus Theoretical Electron Temperature for Gas Discharge Noise Sources"

RONALD E. GUENTZLER

***Abstract*—Previously published microwave noise temperatures from wall-contained argon discharges at 200 mA dc are compared with the electron temperatures predicted by von Engel and Steenbeck. A good agreement between data and theory results if the ionization efficiency is modified to account for stepwise ionization.**

In the above paper,[1] Olson presented a comprehensive study of measured microwave noise temperatures of commercial-gas discharge noise sources. A poor correlation existed between the theoretical electron temperatures as predicted by von Engel and Steenbeck [1, p. 242] and most of the measured noise temperatures. The data were from both wall-contained and constricted discharges.

While studying Olson's figure 2,[1] it became apparent that there was a definite trend in the data from the wall-contained discharges (points 1, 7, and 11–16). This trend was reinforced when the wall-contained data published by Denson and Halford [2, fig. 2] were plotted along with Olson's data. Although all the wall-contained data disagreed with the theoretical curve, they yielded a least squares fit with $\sigma = 1.25$ percent (0.05 dB).

Further investigation of the wall-contained data revealed that if the von Engel and Steenbeck constant c were multiplied by 2, the measured noise temperatures would fit the modified von Engel and Steenbeck curve with $\sigma = 1.75$ percent (0.07 dB), with the furthest point being only 3.5 percent (0.14 dB) in error. This is shown in Fig. 1. (Although the original Olson data points 8 and 9 fall in the area covered in Fig. 1 they are from constricted discharges and are not shown here.)

The von Engel and Steenbeck constant c is obtained from the formula [1, p. 242]

$$c = [aV_i^{1/2}/k^+p]^{1/2}$$

where a is the ionization efficiency, V_i is the ionization potential, k^+ is the positive ion mobility, and p is the pressure.

Traditionally, c (and thus the theoretical electron temperature) was calculated using a value of a appropriate for *direct ionization* by electron impact; however, *stepwise ionization* takes place at the pressures and currents used in the tubes studied [1, p. 244], [3, p. 29]. The excited atoms have a larger ionization cross section than ground-state atoms and can be ionized by the more numerous lower energy electrons; therefore, the ionization efficiency a can be 4 times greater than the value used under the assumption of direct ioniza-

Manuscript received July 18, 1973; revised October 10, 1973.
The author is with the Department of Electrical Engineering, Ohio Northern University, Ada, Ohio 45810.

[1] K. W. Olson, *IEEE Trans. Microwave Theory Tech.* (*Special Issue on Noise*), vol. MTT-16, pp. 640–645, Sept. 1968.